PHASE ERRORS IN SURFACE ACOUSTIC WAVE DEVICES UNDER ROTATION

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Surface acoustic wave (SAW) devices are widely applied in moving objects for carrying out of coordinates, velocities, accelerations and electromagnetic signal parameters accurate measurements. It's known that the rotation influences upon a phase of resonance structure mechanical oscillations and on ultrasonic wave velocity [1]. SAW device rotation leads to SAW phase errors. In order to meet the demand of phase stability in high accuracy modern SAW devices it's required to carry out exact numerical analysis of phase errors arising in SAW devices under rotation.

In order to calculate phase change in SAW device under rotation it's necessary to introduce Coriolis force into the piezo-electric medium movement equations and into the boundary conditions on surface of piezoelectric crystal.

The piezoelectric medium movement equations can be expressed as [2]:

$$\rho \ddot{u}_i = c_{ijlm} \partial_j \partial_l u_m + e_{lij} \partial_j \partial_l \varphi , \tag{1}$$

where ρ is the crystal density, u_i are the elastic displacement components, c_{ijlm} are the elastic constants, e_{lij} are the pie-zoelectric constants, φ is the scalar electric potential, ∂_i is the differentiation operator. In these equations, the summation convention for repeated indices is employed and the tensors are expressed by Einstein's expression.

The Coriolis force is [3]:

$$\vec{K} = 2\rho \left[\vec{v}\,\vec{\Omega} \right] ,$$

and it's components take the form:

$$K_m = 2\rho \varepsilon_{mjk} \dot{u}_j \Omega_k , \tag{2}$$

where K_m are the Coriolis force components, ε_{mjk} are the Levi-civita tensor components, Ω_k are the angular velocity vector components.

Then, the modified piezoelectric medium movement equations under rotation can be written as:

$$\rho \ddot{u}_i = c_{ijlm} \partial_j \partial_l u_m + e_{lij} \partial_j \partial_l \varphi + 2\rho \varepsilon_{ijk} \dot{u}_j \Omega_k . \tag{3}$$

For the plane harmonic wave the elastic displacement components and the scalar electric potential can be expressed in the forms:

$$\left\{ u_i , \varphi \right\} = \left\{ \overline{u}_i , \overline{\varphi} \right\} \exp\left(i\omega t - ik_m x_m \right). \tag{4}$$

Substituting (4) into (3), we obtain the following four linear equations for the four undetermined constants $\overline{u}_i$ and $\overline{\varphi}$:

$$\left(\rho\omega^2 \delta_{im} - c_{ijlm} k_j k_l + 2i\rho\omega\varepsilon_{imj} \Omega_j \right) \overline{u}_m - e_{lij} k_j k_l \overline{\varphi} = 0 . \tag{5}$$

The mechanic boundary conditions on surface of piezoelectric crystal can be expressed as [2]:

$$\sigma_{i3} = 0, \tag{6}$$

where σ_{ij} is the Cauchy stress tensor components.

As Coriolis force acts upon the moving medium particles located on crystal surface $(x_3 = 0)$ on the part of all the moving medium particles under crystal surface $(x_3 < 0)$ the conditions of free surface equilibrium can be written as:

$$\sigma_{i3} + \int_{-\infty}^{0} F_{Cor,i}(x_3)dx_3 = 0. \tag{7}$$

The four-component SAW elastic displacement components can be expressed as:

$$U_j = \sum_{\nu=1}^{4} u_j^{(\nu)} = \sum_{\nu=1}^{4} P_{j\nu}\varphi_\nu ,$$

where U_j are the SAW elastic displacement components, $u_j^{(\nu)}$ are the SAW partial mode elastic displacement components, $P_{j\nu}$ are the polarization coefficients, φ_ν are the partial scalar electric potentials.

The mechanic boundary conditions on surface of piezoelectric crystal under rotation can be expressed as:

$$\sum_{\nu=1}^{4}(c_{i3lm}P_{m\nu} + e_{li3})\left(-ik_l^{(\nu)}\right)\varphi_\nu + \int_{-\infty}^{0} F_{Cor,i}(x_3)dx_3 = 0. \tag{8}$$

Coriolis force acting on particles which participate in the four-component SAW movement can be expressed as:

$$F_{Cor,i} = 2\rho\varepsilon_{ijk}\dot{U}_j\Omega_k = i2\rho\omega\varepsilon_{ijk}\Omega_k\sum_{\nu=1}^{4} u_j^{(\nu)} = i2\rho\omega\varepsilon_{ijk}\Omega_k\sum_{\nu=1}^{4} P_{j\nu}\varphi_\nu .$$

Then

$$\int_{-\infty}^{0} F_{Cos}(x_3)dx_3 = -2\rho\omega\varepsilon_{ijk}\Omega_k\sum_{\nu=1}^{4}\frac{P_{j\nu}}{k_3^{(\nu)}}\varphi_\nu ,$$

and on the basis of (8) we can write:

$$\sigma_{i3} = -i\sum_{\nu=1}^{4}(c_{i3lm}P_{m\nu} + e_{li3})k_l^{(\nu)}\varphi_\nu - 2\rho\omega\varepsilon_{ijk}\Omega_k\sum_{\nu=1}^{4}\frac{P_{j\nu}}{k_3^{(\nu)}}\varphi_\nu = 0, \tag{9}$$

Then, the mechanic boundary conditions on surface of piezoelectric crystal under rotation (8) can be written in the form:

$$\sum_{\nu=1}^{4}\left[(c_{i3lm}P_{m\nu} + e_{li3})k_l^{(\nu)} - i2\rho\omega\varepsilon_{ijk}\Omega_k\frac{P_{j\nu}}{k_3^{(\nu)}}\right]\varphi_\nu = 0. \tag{10}$$

The strict decision of a boundary problem for the Maxwell equation $\left(div\vec{D}=0\right)$ and the piezoelectric medium movement equations (3) with boundary conditions for the electric field and the elastic stress is obtained based on invariant algorithm [4], including Coriolis force effect.

Calculations have been carried out for phase errors in ST-cut quartz-based SAW device operating at 30 MHz under rotation with angular velocity 360º/s on the propagation path 100 wavelengths. We chose the frequency 30 MHz because it's the lowest operating frequency of SAW devices. Phase change effect is inversely proportional to operation frequency and it's essentially proportional to angular velocity and propagation path. Fig. 1. show the results of the calculation for the angular velocity vector orientation across the device base planes.

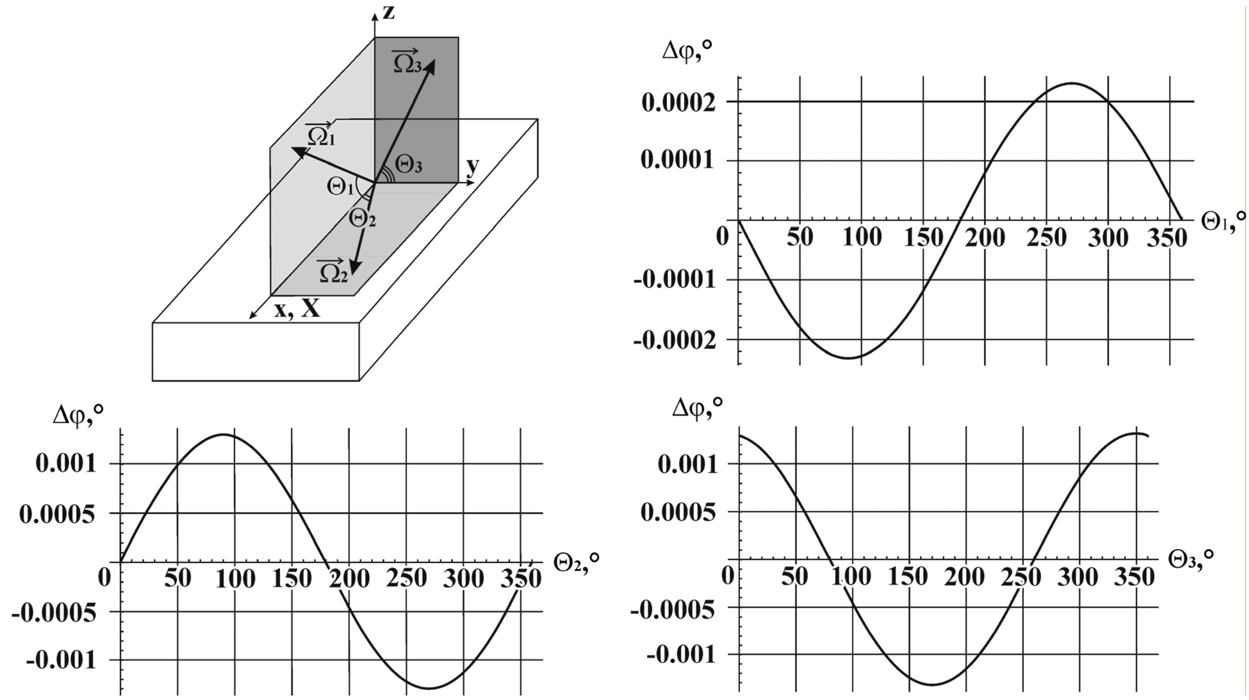

Fig. 1. Phase errors in ST-cut quartz-based SAW device operating at 30 MHz under rotation with angular velocity 360º/s on the propagation path 100 wavelengths for the angular velocity vector orientation across the device base planes

We can conclude, that phase errors arising in SAW devices under rotation with angular velocity 360º/s can achieve the value of 10^{-3}. Whereas a phase stability requirement of a SAW device is 10^{-6}. Therefore in order to prevent such phase errors in high-accuracy modern SAW devices it's necessary to use high frequencies and to choose the spatial orientation of SAW device in moving object.

References:

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